

DETAILED EXPLANATION – CLOCKS

Both the hands start their journey at 12:00 AM with the hour hand moving 5.5° faster than hour hand per minute.

To find the number of times minute hand and hour hand are at 90° :

At 12:00 AM, angle between the hands is 0° .

The first time 90° is formed in a day after $(90^\circ/5.5^\circ)$ minutes = $\frac{180}{11}$ minutes past 12:00 AM

i.e. at $12:16\frac{4}{11}$ AM

The second time 90° is formed in a day when angle between hands is 270° i.e. after $(270^\circ/5.5^\circ) = \frac{540}{11}$ minutes past 12:00 AM.

The second time 90° is formed at $12:49\frac{1}{11}$ AM.

Thus, right angle is formed between the hands when the time is $(n \times \frac{180}{11})$ minutes past 12 where n is an odd number.

Total number of minutes in a day = $24 \times 60 = 1440$ minutes

Maximum value n can take = $1440 / (\frac{180}{11}) = 88$

There can be 88 values of n, where we have to consider only the odd values.

Odd values of n = $88/2 = 44$.

Hence, both hands form 90° , 44 times in a day.

Both the hands are opposite when angle between the hands is 180° .

The first time 180° is formed in a day after $(180^\circ/5.5^\circ)$ minutes = $\frac{360}{11}$ minutes past 12:00 AM
= $2 \times \frac{180}{11}$ minutes past 12:00 AM
i.e. at $12:32\frac{8}{11}$ AM

The second time 180° is formed in a day after $((360+180)^\circ/5.5^\circ)$ minutes = $\frac{1080}{11}$ minutes past 12:00 AM
= $6 \times \frac{180}{11}$ minutes past 12:00
i.e. at $1:38\frac{2}{11}$ AM

Thus, hands at 180° , $(n \times \frac{180}{11})$ minutes past 12 where n is 2, 6, 10 ... 86. (An even number not divisible by 4).
[Values of n have to be less than 88]

Both the hands coincide when angle between the hands is 0° or 360° .

The first time 360° is formed in a day at 12:00 AM.

The second time 360° is formed in a day after $(360^\circ/5.5^\circ)$ minutes = $\frac{720}{11}$ minutes past 12:00 AM

$$= 4 \times \frac{180}{11} \text{ minutes past 12:00 AM}$$

i.e. at $1:05\frac{5}{11}$ AM

The third time 180° is formed in a day after $((360+180)^\circ/5.5^\circ)$ minutes = $\frac{1440}{11}$ minutes past 12:00 AM

$$= 8 \times \frac{180}{11} \text{ minutes past 12:00}$$

i.e. at $2:10\frac{10}{11}$ AM

Thus, hands coincide, $(n \times \frac{180}{11})$ minutes past 12 where n is 0, 4, 8, 12 ... 84. (An even number divisible by 4).

[Values of n have to be less than 88 as at n= 88, next day starts]